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On the investigation of Cerenkov radiation produced by coupling of unstable plasma waves within the ionosphere

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Electrostatic and Electromagnetic interactions have been taken into account to explain the generation of Cerenkov Mode along with the Doppler and Anomalous Doppler Modes within the equatorial E-region of the ionosphere. The results may be used to investigate the nature of irregularities produced by coupling of unstable plasma waves within such a region.

1. Introduction

The phenomena of scattering of electromagnetic waves within the E-region of the ionosphere due to electron number-density fluctuation associated with the equatorial electrojet have been investigated during the last few years by Bowles *et al* (1963, 1965), Farley (1963), Blasley (1965) Dougherty & Farley (1967), Whitehead (1968) and others. The aspect sensitive echoes due to scattering are generally explained by the theory of two-stream instability after Dougherty and Farley (1967). The equatorial scatter during weak-electroject produces various asymmetric frequency spectra which need still a complete explanation. In some cases, Helliwell's theory (1967) has been utilised to study these spectra (Dowden 1971). In this presentation, the electrostatic and electromagnetic interactions within the E-region of the ionosphere have been considered to investigate the VLF scatter phenomenon utilising ideas of Dougherty and Farley. Under appropriate circumstances, this method yields some of the frequency spectra which are characteristics of Cerenkov type of radiation propagating along with the normal and anomalous Doppler modes of propagation.

2. Mathematical Formulation

In this presentation, the system has been characterised through Maxwell's and Euler equations (Clauser 1960) and for this, the linearised equations for small perturbations can be written as

$$\left.\begin{aligned}
&\nabla\times\boldsymbol{E}_1 = -\frac{\partial \boldsymbol{B}_1}{\partial t}\\
&\frac{1}{\mu_0}\nabla\times\boldsymbol{B}_1 = \boldsymbol{J}_1+\epsilon_0\frac{\partial \boldsymbol{E}_1}{\partial t}\\
&\nabla\cdot\boldsymbol{E}_1 = \rho_1/\epsilon_0\\
&\nabla\cdot\boldsymbol{B}_1 = 0\\
&\frac{\partial\rho_1}{\partial t}+\nabla\cdot\boldsymbol{J}_1 = 0
\end{aligned}\right\} \quad \dots (1)$$

$$\frac{\partial \boldsymbol{v}_1}{\partial t}+(\boldsymbol{v}_0\cdot\nabla)\boldsymbol{v}_1+(\boldsymbol{v}_1\cdot\nabla)\boldsymbol{v}_0 = -\frac{e}{m}(\boldsymbol{E}_1+\boldsymbol{v}_1\times\boldsymbol{B}_0+\boldsymbol{v}_0\times\boldsymbol{B}_1) \quad \dots (2)$$

and $$\boldsymbol{J}_1 = \rho_0\boldsymbol{v}_1+\rho_1\boldsymbol{v}_0 \quad \dots (3)$$

where $$\boldsymbol{E} = \boldsymbol{E}_0+\boldsymbol{E}_1,\ \boldsymbol{B}_1 = \boldsymbol{B}_0+\boldsymbol{B}_1 \text{ etc.}$$

The terms which are quadratic in the perturbation have been neglected. We assume that $(\boldsymbol{v}_0\times\boldsymbol{B}_1)\ll\boldsymbol{E}_1$ which is in accordance with the assumption of the non-relativistic electrons within the ionospheric plasma. Of course, this approach leads to the description of space-charge waves. Under the circumtances thes wave equations can be written as (Clauser 1960, Khan 1972),

$$\left(\nabla^2-\frac{\omega_p{}^2}{c^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{A}_1 = -\mu_0\boldsymbol{J}_1$$

$$\left(\nabla^2-\frac{1}{v_0{}^2}\frac{\partial^2}{\partial t^2}-\frac{\omega_p{}^2}{v_0{}^2}\right)n_1 = \frac{\omega^2{}_p\rho_1}{ev_0{}^2}$$

$$\left(\frac{\partial^2}{\partial t^2}+\omega_p{}^2\right)\left(\nabla^2-\frac{\omega_p{}^2}{c^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi_1 = -\epsilon_0{}^{-1}\frac{\partial^2\rho_1}{\partial t^2}$$

where, $\boldsymbol{A}_1$ = vector potential of the electromagnetic field
ϕ_1 = scalar potential of the electromagnetic field
n_1 = fluctuation in electron density.

The corresponding expressions for the charge, velocity and current density can be found out (Clauser 1960). The Fourier-transform expressions of $\boldsymbol{J}_1$ and ρ_1 yield

$$\boldsymbol{J}_1(k,\omega) = [c_1\delta\{kv_0-(\omega-\omega_p')\}+c_2\delta\{kv_0-(\omega+\omega_p')\}] \quad \dots (4)$$

$$\rho_1(k,\omega) = \left[\left(1-\frac{\omega_p'}{\omega}\right) c_1\delta\{kv_0-(\omega-\omega_p')\} + \left(1+\frac{\omega_p'}{\omega}\right) c_2\delta\{kv_0-(\omega+\omega_p')\}\right] \quad \ldots (5)$$

ω_p' = average Langmuir frequency of particles.

With the above expressions, Fourier transform of the vector and scalar potentials can be obtained and through the argument of the δ-function [Ginsburg and Eidman (1959)] the conditions of radiation with three different discrete frequency spectra can be deduced from the relation given by

$$\omega = \frac{\omega_p'}{1-\frac{v_0}{v_\phi}\cos\theta} \quad \ldots (6)$$

where v_ϕ is the phase velocity of plasma waves and θ is the angle between the wave vector and velocity vector.

From (6) the following case will be followed :

(a) when $\frac{v_0}{v_\phi}\cos\theta = 1$, $\omega = \infty$; ... (7)

this yields Cerenkov Mode.

(b) when $\frac{v_0}{v_\phi}\cos\theta < 1$, $\omega = \dfrac{\omega_p'}{1-\frac{v_0}{v_\phi}\cos\theta}$; ... (8)

gives Normal Doppler Mode;

and

(c) when $\frac{v_0}{v_\phi}\cos\theta > 1$, $\omega = \dfrac{\omega_p'}{\frac{v_0}{v_\phi}\cos\theta-1}$; ... (9)

this will yield Anomalous Doppler Mode

3. Discussions

The modes given by eqs. (7), (8) and (9) are produced by the coupling of unstable plasma waves which are characteristics of the physical conditions of the E-region of the ionosphere during equatorial electrojet.

Travelling ionospheric disturbances have been investigated by various workers (Hines 1960, Georges 1968) through which different properties of the ionosphere are studied. The usual travelling wave distrurbances from Faraday rotation observations from the radio becon satellite $BE=B$, inclination 80°, observed at Urbana, Illinois (40.1°N, 87.2°W); Danville, Illinois (40.1°N, 87.6°W); London, Ontario (43.0°N, 81.3°W) and Hamilton, Massachusetts (42.6°N, 70.8°W), have been investigated by Rao, Lyon and Klobuchar (1969), from which interpretations for various frequency spectrum fluctuations are gathered which show considerable ambiguity in the velocity of propagation of various wave-fronts and in the corresponding phase velocities. On the other hand, studies on acoustic waves in the ionosphere do not support fully the existence of such characteristic fluctuations. Another way of explaining the various modes of propagation is through internal atmospheric gravity waves which are supposed to introduce the wind in the E-region of the ionosphere and travelling wave disturbances. But theoretical investigation on gravity waves are not capable of explaining the entire band of frequencies which are obtained from experimental findings by Kohl & King (1967); Klostermeyer (1969).

Applying Helliwell's criterion, different energy spectra and their structure near the equatorial ionosphere are recognised as discrete VLF emissions. The existence of band limited noise (hiss) has also been reported by Dowden (1971). Within the equatorial E-region of the ionosphere hiss appears to be produced by convective instability, while discrete emissions are produced due to non-convective instabilities. It is also suggested that the radiative instability (transverse or electroncyclotron) may be the same for both the types, since it is explained that cyclo-tron-resonance amplification by electrons having energy and pitch angle distribution consistent with satellite measurements could produce both the hiss amplitude and discrete spectrum observed.

The anomaly in the interpretation of the observed spectrum from various angles may be looked upon by taking into account both the electrostatic and other dissipative processes within the equatorial E-region of the ionosphere. The discrete frequency spectra through such analysis have been obtained which are expressed through (7), (8) and (9). The observed anomaly may be explained by this method.

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